Stabilization of nD behaviors

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NDS09

Talk outline

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- Once more … Behavioral systems

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- Again... Behavioral interconnection and control

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- Stability of behavioral systems

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- Stability of behavioral systems
- Stabilization by full interconnection

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- Once more … Behavioral systems
- Again… Behavioral interconnection and control
- Stability of behavioral systems
- Stabilization by full interconnection
- Not yet… Stabilization by partial interconnection

Behavioral systems

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Behavioral system $\Sigma = (\mathbb{Z}^n, \mathbb{R}^{\mathtt{w}}, \mathcal{B}_w)$

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Behavior descriptions

$R(\sigma_1, \ldots, \sigma_n) w = 0 \quad \mathcal{B} = \ker R$ kernel

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$w = M(\sigma_1, \ldots, \sigma_n)\, a \quad \mathcal{B} = im\, M$ image $\leftrightsquigarrow$ controllable behaviors

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Latent variable elimination

$R(\sigma_1, \ldots, \sigma_n)w = M(\sigma_1, \ldots, \sigma_n)\, a \longleftrightarrow LRw = 0$ L MLA of R

$im\, M = \ker L$

Behavioral systems

$\mathcal{B} = \ker R$ is characterized by the set of equations satisfied by all its elements $\sim$ $Mod(\mathcal{B})$

$$Mod(\mathcal{B}_1 \cap \mathcal{B}_2) = Mod(\mathcal{B}_1) + Mod(\mathcal{B}_2)$$
$$Mod(\mathcal{B}_1 + \mathcal{B}_2) = Mod(\mathcal{B}_1) \cap Mod(\mathcal{B}_2)$$

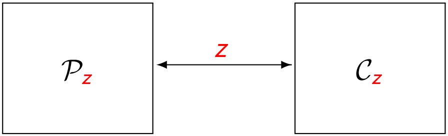
Autonomous-controllable decomposition

$$\mathcal{B} = \mathcal{B}^{controllable} + \mathcal{B}^{autonomous}$$

Behavioral control by full interconnection

Control setting

$\mathcal{P}_z$ $\rightsquigarrow$ plant $\rightsquigarrow$ behavior to be controlled
$\mathcal{C}_z$ $\rightsquigarrow$ full controller



$\mathcal{P}_z \cap \mathcal{C}_z$ $\rightsquigarrow$ plant-controller interconnection $\rightsquigarrow$ controlled behavior

Behavioral control by full interconnection

Implementation

$\mathcal{D}_z$ - control objective - is implementable from $\mathcal{P}_z$ by full interconnection if a full controller $\mathcal{C}_z$ exists such that

$$\boxed{\mathcal{P}_z \cap \mathcal{C}_z = \mathcal{D}_z}$$

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$$\mathcal{P}_z = \ker H; \quad \mathcal{C}_z = \ker K; \;\; \mathcal{D}_z = \ker D \qquad \boxed{\ker \begin{bmatrix} H \\ K \end{bmatrix} = \ker D}$$

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$$\boxed{\mathrm{Mod}(\mathcal{P}_z) + \mathrm{Mod}(\mathcal{C}_z) = \mathrm{Mod}(\mathcal{D}_z)}$$

NSC for implementation

$\mathcal{D}_z \subset \mathcal{P}_z$

Behavioral control by full interconnection

Regularity

The interconnection $\mathcal{P}_z \cap \mathcal{C}_z$ is regular if

$$\boxed{\mathrm{Mod}(\mathcal{P}_z) \cap \mathrm{Mod}(\mathcal{C}_z) = \{0\}}$$

Meaning: The plant and the controller share no (non-trivial) equations

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$\mathcal{P}_z = \ker H \quad \mathcal{C}_z = \ker K \quad \boxed{\mathrm{rank} \begin{bmatrix} H \\ K \end{bmatrix} = \mathrm{rank}\ H + \mathrm{rank}\ K}$

Behavioral control by full interconnection

Regular implementation problem

Given: $\mathcal{P}_z = \ker H$ and $\mathcal{D}_z = \ker D$

Find: $\mathcal{C}_z = \ker K$ such that its interconnection with $\mathcal{P}_z$ is regular and yields $\mathcal{D}_z$

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$$\mathrm{Mod}(\mathcal{D}_z) = \mathrm{Mod}(\mathcal{P}_z) \oplus \mathrm{Mod}(\mathcal{C}_z)$$

$$\ker \begin{bmatrix} H \\ K \end{bmatrix} = \ker D; \ \mathrm{rank} \begin{bmatrix} H \\ K \end{bmatrix} = \mathrm{rank}\ H + \mathrm{rank}\ K$$

Behavioral control by full interconnection

Example

$$x(k+1) = Ax(k) + Bu(k) \qquad \underbrace{\begin{bmatrix} \sigma I - A & -B \end{bmatrix}}_{H} \begin{bmatrix} x \\ u \end{bmatrix} = 0$$

$$u = Fx \qquad \underbrace{\begin{bmatrix} -F & I \end{bmatrix}}_{K} \begin{bmatrix} x \\ u \end{bmatrix} = 0$$

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Plant $\quad Hz = 0 \qquad\qquad$ Controller $\quad Kz = 0$

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Regular interconnection

Behavioral control by full interconnection

Solution of the regular implementation problem - 1D case

Theorem: Given $\mathcal{P}_z$ and $\mathcal{D}_z \subset \mathcal{P}_z$

The regular implementation problem is solvable
$\Leftrightarrow$ $\mathcal{P}_z / \mathcal{D}_z$ is controllable
$\Leftrightarrow$ $\mathcal{P}_z = \mathcal{P}_z^{controllable} + \mathcal{D}_z$.

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Controller construction

$\mathcal{D}_z = \ker D$; $\mathcal{P}_z = \ker MD$; D and M full row rank

$\mathcal{P}_z/\mathcal{D}_z \backsimeq \ker M$

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$\mathcal{C}_z = \ker ND$ is a regular controller

Behavioral control by full interconnection

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Theorem: Given $\mathcal{P}_z$ and $\mathcal{D}_z \subset \mathcal{P}_z$

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$\mathcal{D}_z = \ker D$; $\quad \mathcal{P}_z = \ker MD$;

${}^{\mathcal{P}_z}/_{\mathcal{D}_z} \backsimeq \ker \begin{bmatrix} M \\ E \end{bmatrix}$ and ${}^{\mathcal{A}^z}/_{\mathcal{D}_z} \backsimeq \ker E$ with E MLA of D

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$$\ker \begin{bmatrix} M \\ E \end{bmatrix} \oplus \underbrace{\ker K}_{\text{regular controller}} = \ker E$$

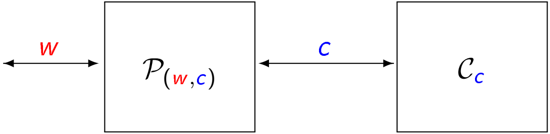
(Lomadze-Zerz; Bisiacco-Valcher)

Behavioral control by partial interconnection

Control setting

$z = (w, c)$: c - control variables; w - variables to be controlled

$\mathcal{P}_{(w,c)} \rightsquigarrow$ plant $\mathcal{C}_c \rightsquigarrow$ (partial) controller



$\mathcal{P}_{(w,c)} \cap \mathcal{C}^*_{(w,c)} \rightsquigarrow$ plant-controller interconnection $\rightsquigarrow$ full controlled behavior

Behavioral control by partial interconnection

Implementation

$\mathcal{D}_w$ - control objective - is implementable from $\mathcal{P}_{(w,c)}$ by partial interconnection if a (partial) controller $\mathcal{C}_c$ exists such that

$$\boxed{\Pi_w(\mathcal{P}_{(w,c)} \cap \mathcal{C}^*_{(w,c)}) = \mathcal{D}_w}$$

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$$\mathcal{P}_{(w,c)} : \quad Rw = Mc; \quad \mathcal{D}_w : \quad Dw = 0; \quad \mathcal{C}_c : \quad Kc = 0$$

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$$\begin{bmatrix} R \\ 0 \end{bmatrix} w = \begin{bmatrix} M \\ K \end{bmatrix} c \;\longrightarrow \textit{elimination of } c \longrightarrow \; Dw = 0$$

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$$L \text{ MLA of } \begin{bmatrix} M \\ K \end{bmatrix} \qquad L \begin{bmatrix} R \\ 0 \end{bmatrix} w = 0 \iff Dw = 0$$

Behavioral control by partial interconnection

Implementation

NSC for implementation

$\mathcal{P}_{(w,0)} := \left\{ w \mid (w,0) \in \mathcal{P}_{(w,c)} \right\}$ hidden w-behavior;
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$\mathcal{D}_w \quad FR\,w = 0$

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Behavioral control by partial interconnection

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$$\operatorname{rank} \begin{bmatrix} R & -M \\ 0 & K \end{bmatrix} = \operatorname{rank} \begin{bmatrix} R & -M \end{bmatrix} + \operatorname{rank} \begin{bmatrix} 0 & K \end{bmatrix}$$

Behavioral control by partial interconnection

Regular implementation problem

Given: $\mathcal{P}_{(w,c)} = \ker[R \quad -M]$ and $\mathcal{D}_w = \ker D$

Find: $\mathcal{C}_c = \ker K$ such that its (partial) interconnection with $\mathcal{P}_{(w,c)}$ is regular and yields $\mathcal{D}_w$

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More difficult than full interconnection!

Behavioral control by partial interconnection

Example

$$\left\{ \begin{array}{lcl} x(k+1) & = & Ax(k) + Bu(k) \\ y(k) & = & Cx(k) + Du(k) \end{array} \right. \qquad \underbrace{\begin{bmatrix} \sigma I - A \\ C \end{bmatrix}}_{R} \underbrace{x}_{w} = \underbrace{\begin{bmatrix} B & 0 \\ -D & I \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} u \\ y \end{bmatrix}}_{c}$$

$$y = Fu \qquad \underbrace{\begin{bmatrix} -I & F \end{bmatrix}}_{K} \begin{bmatrix} u \\ y \end{bmatrix} = 0 \qquad \text{partial controller}$$

Behavioral control by partial interconnection
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Plant-controller interconnection
$$\begin{bmatrix} \sigma I - A & -B & 0 \\ C & -D & I \\ 0 & -I & F \end{bmatrix} \begin{bmatrix} x \\ u \\ y \end{bmatrix} = 0$$

regular if $(I - FD)$ nonsingular

Behavioral control by partial interconnection

Example

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases} \qquad \underbrace{\begin{bmatrix} \sigma I - A \\ C \end{bmatrix}}_{R} \underbrace{x}_{w} = \underbrace{\begin{bmatrix} B & 0 \\ -D & I \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} u \\ y \end{bmatrix}}_{c}$$

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Plant-controller interconnection

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regular if $(I - FD)$ nonsingular

Controlled behavior $\longrightarrow$ corresponding x-behavior

Behavioral control by partial interconnection

Solution of the regular implementation problem - 1D case

Theorem [Belur-Trentelman, 2002]

$\mathcal{D}_w$ implementable from $\mathcal{P}_{(w,c)}$ by regular partial interconnection
$\Longleftrightarrow$
$\mathcal{P}_{(w,0)} \subset \mathcal{D}_w$ and
$\mathcal{D}_w$ implementable from $\mathcal{P}_w$ by regular full interconnection

Behavioral control by partial interconnection

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Controller construction

$$\mathcal{P}_{(w,c)} \qquad R\,w = M\,c$$

$$\mathcal{P}_w \qquad G F R\,w = 0$$

$$\mathcal{D}_w \qquad F R\,w = 0$$

Behavioral control by partial interconnection

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Controller construction

$\mathcal{P}_{(w,c)}$	$R\,w = M\,c$
$\mathcal{P}_w$	$G\,F\,R\,w = 0$
$\mathcal{D}_w$	$F\,R\,w = 0$

Regular w-controller	$\mathcal{C}_w$	$K\,F\,R\,w = 0$
Regular partial controller	$\mathcal{C}_c$	$K\,F\,M\,c = 0$

Behavioral control by partial interconnection

Solution of the regular implementation problem - nD case

Unfortunately...

Behavioral control by partial interconnection

Solution of the regular implementation problem - nD case

Unfortunately...

$\mathcal{D}_w$ implementable from $\mathcal{P}_{(w,c)}$ by regular partial interconnection

$\not\Rightarrow$

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Behavioral control by partial interconnection

Solution of the regular implementation problem - nD case

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Example

$$\mathcal{P}_{(w,c)} \rightsquigarrow w = \begin{bmatrix} \sigma_2 - 1 \\ 1 - \sigma_1 \end{bmatrix} c \qquad \mathcal{P}_w = \ker[\sigma_1 - 1 \quad \sigma_2 - 1]$$

$$\mathcal{D}_w = \{0\} \rightsquigarrow w = 0$$

$\mathcal{C}_c = \ker 1 \rightsquigarrow c = 0 \rightsquigarrow$ regular controller

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The partial implementation problem can still be reformulated as a full implementation problem

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Plant $\rightsquigarrow \mathcal{P}_{(w,c)} \rightsquigarrow Rw = Mc \rightsquigarrow$ elimination of $w \rightsquigarrow \mathcal{P}_c$

Control objective $\rightsquigarrow \mathcal{D}_w \rightsquigarrow Dw = 0$

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Associated canonical controller

$$\mathcal{C}_c^{can} := \{c \mid \exists\ w \in \mathcal{D}_w \text{ s.t. } (w,c) \in \mathcal{P}_{(w,c)}\} = \Pi_c(D^*_{(w,c)} \cap \mathcal{P}_{(w,c)}\}$$

$$\left\{ \begin{array}{rcl} Dw & = & 0 \\ Rw & = & Mc \end{array} \right. \rightsquigarrow \text{elimination of } w \rightsquigarrow K_c^{can} c = 0$$

Behavioral control by partial interconnection

Solution of the regular implementation problem - nD case

Theorem [Rocha,2005]
Let $\mathcal{D}_w$ be implementable from $\mathcal{P}_{(w,c)}$ by partial interconnection.
Then
$\mathcal{D}_w$ is regularly implementable $\Leftrightarrow$ $\mathcal{C}_c^{can}$ is regularly implementable from $\mathcal{P}_c$ by full interconnection

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$\mathcal{C}_c$ regular controller that implements $\mathcal{C}_c^{can}$ from $\mathcal{P}_c$ by full interconnection
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But: Full and partial implementation problems have different control objectives...

Behavioral control by partial interconnection
Example

$$\mathcal{P}_{(w,c)} \rightsquigarrow w = \begin{bmatrix} \sigma_2 - 1 \\ 1 - \sigma_1 \end{bmatrix} c; \quad \mathcal{P}_c = \ker 0 \rightsquigarrow \text{ i.e. } c \text{ is free}$$

$$\mathcal{D}_w = \{0\} \rightsquigarrow w = 0$$

$$\mathcal{C}_c^{can} = \ker \begin{bmatrix} \sigma_2 - 1 \\ 1 - \sigma_1 \end{bmatrix}$$

$\mathcal{C}_c = \mathcal{C}_c^{can}$ regularly implements $\mathcal{C}_c^{can}$ from $\mathcal{P}_c$ by full control and hence regularly implements $\mathcal{D}_w$ from $\mathcal{P}_{(w,c)}$ (by partial control)

Summary

nD control

	Full interconnection	Partial interconnection
Plant	$\mathcal{P}_z$	$\mathcal{P}_{(w,c)}$
Control objective	$\mathcal{D}_z$	$\mathcal{D}_w$
Implementation	$D_z \subset \mathcal{P}_z$	$\mathcal{P}_{(w,0)} \subset \mathcal{D}_w \subset \mathcal{P}_w$
Regularity	${}^{\mathcal{P}_z}/{}_{\mathcal{D}_z}$ direct summand of ${}^{\mathcal{A}^z}/{}_{\mathcal{D}_z}$	$\mathcal{C}_c^{can}$ reg. impl. from $\mathcal{P}_c$ by full intercon.
Regular controller	$C_z \simeq$ compl. summand	$\mathcal{C}_c$ regular c-controller

Stability of behavioral systems

1D case

$\mathcal{B}_w$ is stable $\rightsquigarrow$ for all $w \in \mathcal{B}$: $\lim_{k \to +\infty} w(k) = 0$

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Stability cone S

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Our stability notion

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$\mathcal{B}_w$ is stable

For all $w \in \mathcal{B}_w$ and for all elements $\overrightarrow{d}$ in S: $\lim_{k \to +\infty} w(k\overrightarrow{d}) = 0$

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Valcher - uses stability cones, but only some trajectories are required to vanish asymptotically

Oberst-Scheicher - consider an input-output framework

Stability of behavioral systems

Characterization

Remark

$$\mathcal{B}_w = \ker R(\sigma_1, \ldots, \sigma_n) \text{ stable} \quad \Rightarrow \quad \mathcal{B}_w \text{ finite dimensional (strongly autonomous)}$$

$$\Leftrightarrow \quad R(s_1, \ldots, s_n) \text{ has fcr and finite \# of zeros}$$

Theorem

$$\mathcal{B}_w = \ker R(\sigma_1, \ldots, \sigma_n) \text{ stable} \quad \Leftrightarrow \quad \boxed{\begin{array}{l} R(s_1, \ldots, s_n) \text{ has fcr, finite \# of zeros} \\ (d_1, \ldots, d_n) \in S;\ (\lambda_1, \ldots, \lambda_n) \text{ zero of } R \\ \left|\lambda_1^{d_1} \cdots \lambda_n^{d_n}\right| < 1 \end{array}}$$

Stabilization by full interconnection

Problem statement

Given: a plant $\mathcal{P}_w$

Find: a regular controller $\mathcal{C}_w$ such that

$\mathcal{P}_w \cap \mathcal{C}_w$ is a regular interconnection $\boxed{\mathcal{P}_w \cap_{reg} \mathcal{C}_w}$

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If this problem is solvable,

$\mathcal{P}_w$ is said to be stabilizable (by full control).

Stabilization by full interconnection

1D case [Willems]

Theorem

$\mathcal{P}_w$ stabilizable (by full control)

$\iff$

$\mathcal{P}_w = \mathcal{P}_w^{controllable} \oplus \mathcal{P}_w^{autonomous}$ with $\mathcal{P}_w^{autonomous}$ stable

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Cf classical results for state-space systems.

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Cf classical results for state-space systems.

Corollary

$\mathcal{P}_w = \ker R$ stabilizable (by full control)

$\Longleftrightarrow$

$R = QR^{controllable}$ with $\ker R^{controllable} = \mathcal{P}_w^{controllable}$, $\ker Q$ stable

Stabilization by full interconnection

nD case

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Theorem

FD behavior regularly implementable from $\mathcal{P}_w$ $\Rightarrow$ $\mathcal{P}_w^{controllable}$ direct summand of $\mathcal{A}^{\mathtt{w}}$

$$\Updownarrow$$

$\mathcal{P}_w^{controllable}$ rectifiable

$$\Downarrow$$

$$\mathcal{P}_w = \mathcal{P}_w^{controllable} \oplus \mathcal{P}_w^{autonomous}$$

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$$\left.\begin{array}{l}\mathcal{P}_w \text{ stabilizable} \\ \text{by full reg. int.}\end{array}\right| \Leftrightarrow \left|\begin{array}{l}\mathcal{P}_w = \mathcal{P}_w^{controllable} \oplus \mathcal{P}_w^{autonomous} \\ \text{with } \mathcal{P}_w^{autonomous} \text{ stable}\end{array}\right.$$

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$$\mathcal{P}_w = \ker R; \quad R = QR^{controllable} \quad \text{with } \ker Q \text{ stable}$$

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$\exists$ controller $\mathcal{C}_c$ such that $\Pi_w(\mathcal{P}_{(w,c)} \cap_{reg} \mathcal{C}^*{}_{(w,c)})$ is stable

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Theorem [Belur-Trentelman]

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nD case

Also

$\mathcal{P}_{(w,c)}$ stabilizable $\Rightarrow \mathcal{P}_{(w,0)}$ stable

But...

Stabilization by partial interconnection

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The corresponding full interconnection problem is stated in terms the control variables c

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Proposition

$\mathcal{P}_{(w,c)} \rightsquigarrow Rw = Mc$ stabilizable by regular partial intercon.	$\Leftrightarrow$	$\tilde{\mathcal{P}}_c := \mathcal{P}_c/\mathcal{P}_{(0,c)}$ stabilizable by full interconnection with a controller $\tilde{\mathcal{C}}_c$ s.t. $Mod(\tilde{\mathcal{P}}_c) \cap Mod(\tilde{\mathcal{C}}_c) = Mod(im\, M)$
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		$\exists \mathcal{C}_c$: $\tilde{\mathcal{P}}_c \cap \mathcal{C}_c$ is stable; $\tilde{\mathcal{P}}_c + \mathcal{C}_c = im\, M$
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$\tilde{\mathcal{P}}_c$ is an almost direct summand of imM	$\Leftarrow$	$\exists \mathcal{C}_c$: $\tilde{\mathcal{P}}_c \cap \mathcal{C}_c$ FD $\tilde{\mathcal{P}}_c + \mathcal{C}_c = im\, M$

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Valcher, Napp,Oberst

To be continued...

Future work

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Thank you!